Pre-Sliding Friction Control Using the Sliding Mode Controller with Hysteresis Friction Compensator

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Friction phenomenon can be described as two parts, which are the pre-sliding and sliding regions. In the motion of the sliding region, the friction force depends on the velocity of the system and consists of the Coulomb, stick-slip, Streibeck effect and viscous frictions. The friction force in the pre-sliding region, which occurs before the breakaway, depends on the position of the system. In the case of the motion of the friction in the sliding region, the LuGre model describes well the friction phenomenon and is used widely to identify the friction model, but the motion of the friction in the pre-sliding such as hysteresis phenomenon cannot be expressed well. In this paper, a modified friction model for the motion of the friction in the pre-sliding region is suggested which can consider the hysteresis phenomenon as the Preisach model. In order to show the effectiveness of the proposed friction model, the sliding mode controller (SMC) with hysteresis friction compensator is synthesized for a ball-screw servo system.

Key Words: Friction, Preisach Hysteresis Model, Sliding Mode Control, Feedforward Hysteresis Friction Compensator

1. Introduction

Friction, which is a natural characteristic of mechanical systems, has a bad effect on servo dynamic systems. Many researches (Dahl, 1968; Armstrong, 1994; Canudas, 1995) have widely studied to identify the friction phenomenon. The classical friction model describes the static relation between the velocity and the friction force such as Coulomb and viscous frictions. In applications to the high precision position tracking in the low velocity, the compensation of the friction using static friction models often yields poor performance because the internal dynamics of the friction are not considered. In order to describe the static and dynamic motions of the friction, the friction phenomenon should be considered both in the pre-sliding and sliding regions (Dupont, 2002). The friction dynamics in the pre-sliding region were firstly presented by Dahl (1968). In the Dahl model the friction interface between two surfaces is thought as a contact between bristles. This model can account for Coulomb friction, but it cannot present the Streibeck effect. To improve the Dahl model, the LuGre (Lund Grenoble) model was announced by Canudas (1995). The LuGre model con-

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tains the internal dynamics of the friction so that it can explain the stick-slip, Streibeck effect and well known other friction phenomena. The LuGre model, however, cannot describe the friction phenomenon in the pre-sliding region. The one of the limitation of the LuGre model is non-local memory hysteresis phenomenon which has characteristics such as wiping-out and homogenous (Mayergoyz, 1991). The non-local memory hysteresis phenomenon happens frequently in electromagnetic systems (Xiao, 2002; Yu, 2000; Sjostrom, 2000; Cincotti, 1999; Ge, 1995) and has been identified by the Preisach hysteresis model.

In this paper, in order to describe mechanical systems with non-local memory hysteresis phenomenon, the Preisach hysteresis model was utilized. For more effective application of the Preisach hysteresis model, the deformed Dahl model (Awabdy, 1998) was used. The deformed Dahl model represents the hysteresis curve with two different equations which can describe the ascending and descending positions. Using these equations, the transition curve for the Preisach plane was completed and the friction model in the pre-sliding region was accomplished. In order to show the effectiveness of the proposed friction model considered the pre-sliding friction phenomenon, a ball-screw mechanism was applied. The precision position tracking control system was synthesized by the sliding model controller with feedforward friction compensator and the performance of the control system was evaluated through simulations and experiments.

2. Improved Friction Model

2.1 Friction model in the pre-sliding region

The LuGre friction model, which is widely used to represent the friction phenomenon, can be expressed as follows:

$$F=\sigma_0 z+\sigma_1 \dot{z}+\sigma_2 v \tag{1}$$

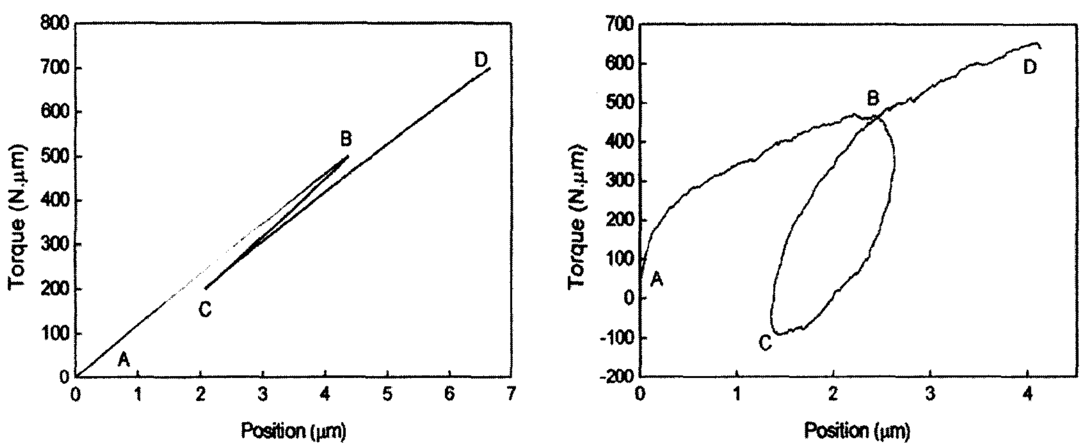
$$\dot{z}=v\left(1-\frac{\sigma_0 z}{g(v)}\operatorname{sgn}(v)\right) \tag{2}$$

$$\sigma_0 g(v)=F_c+(F_s-F_c)\,e^{-(v/v_s)^2} \tag{3}$$

where σ_0 and σ_1 are the stiffness and the viscous friction of the bristles respectively, σ_2 is the viscous friction, z is the position in the pre-sliding region, and v and v_s are the velocity of the position and Streibeck velocity, respectively and F_c and F_s are Coulomb and stick friction forces, respectively. Also, in the steady-state motion the relation between the velocity and the friction force F_{ss} is given by

$$F_{ss}=F_c\operatorname{sgn}(v)+(F_s-F_c)\,e^{-(v/v_s)^2}\operatorname{sgn}(v)+\sigma_2 v \tag{4}$$

In order to verify the friction phenomena, a ball-screw system is considered to experiment.

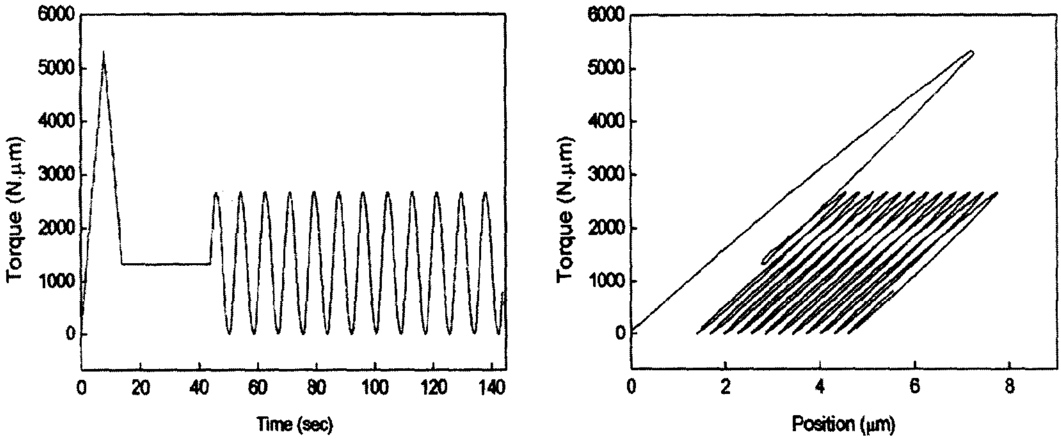


(a) Simulation result (b) Experimental result

Fig. 1 Output torque against position in the LuGre model and real system

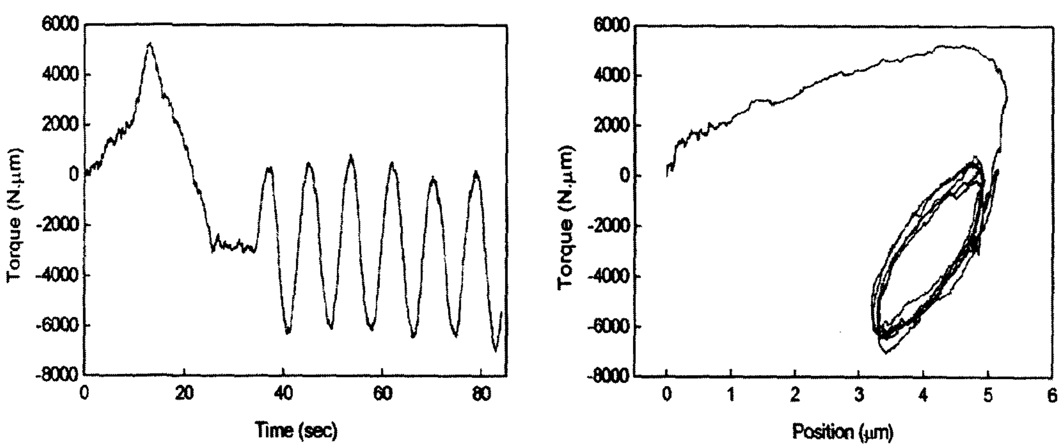
The ball-screw system has the linear guide and works on a DC servomotor with an encoder whose resolution is 40,000 pulse/rev. The torque was measured by the torque sensor of RSL-210 produced by the Lorenz Messtechnik GmbH company (Germany) which was attached between the motor and the ball-screw. The control system was implemented by the LabVIEW system of N.I. company.

Figure 1 shows the simulation result of the LuGre friction model and experimental result of the ball-screw system for the command input which moves forward and backward (A-B-C-B-D) in the pre-sliding region. The closed-loop, which is the non-local memory hysteresis, is shown in Fig. 1(b) when the position is returned to B, but it can not be found in Fig. 1(a). In Fig. 2(a) the friction torque for the oscillated input torque is shown. This input torque ramps up to cause break-away, and then return to a level below that of Coulomb friction. As shown in Fig. 2(b), the friction torque is continuously drifted for the small oscillated input torque. However, in the real system shown in Fig. 3, the elliptical orbits of the friction against the small oscillated input torque were represented.



(a) Command input torque (b) Output torque against position

Fig. 2 Friction drift for the oscillated command input torque in the LuGre model



(a) Control input torque (b) Output torque against position

Fig. 3 Friction drift for the oscillated control input and output torques in the resl system

Compared with the simulation and experimental results for the small oscillated input torque, it is found that the LuGre model should be modified in the pre-sliding region. Thus, we suggest a modified friction model as follows:

$$F_p=f_h(z)+\sigma_1\dot{z}+\sigma_2 v \tag{5}$$

$$\dot{z}=v\left(1-\frac{f_h(z)}{G(v)}\operatorname{sgn}(v)\right) \tag{6}$$

$$G(v)=F_c+(F_s-F_c)e^{-(v/v_s)^2} \tag{7}$$

where $f_h(z)$ is the friction of hysteresis and F_p is the friction force in the proposed friction model. In the proposed friction model, the stiffness term of the LuGre model is substituted by the Preisach model. The supplemented $f_h(z)$ in the pre-sliding region does not act linearly to the unmeasured z and it makes the hysteresis curve describe the non-local memory characteristic. Also, in the sliding region the derivative of z goes to 0 and then the proposed friction can be described as follows:

$$\dot{z}=0,\ v\neq 0 \tag{8}$$

$$1=\frac{f_h(z)}{G(v)}\operatorname{sgn}(v) \tag{9}$$

$$f_h(z)=G(v)\operatorname{sgn}(v) \tag{10}$$

$$\begin{aligned}F_p&=G(v)+\sigma_2 v\\&=[F_c+(F_s-F_c)e^{-(v/v_s)^2}]\operatorname{sgn}(v)+\sigma_2 v\end{aligned} \tag{11}$$

The proposed and LuGre friction models in the steady-state motion are equal which can show in Eqs. (4) and (11).

2.2 Preisach model for the pre-sliding friction

The classical Preisach model for the pre-sliding friction is represented as follows (Mayergoyz, 1991):

$$f_h(z)=\iint_{\alpha\geq\beta}\mu(\alpha,\beta)\,\gamma_{\alpha\beta}z(t)\,d\alpha d\beta \tag{12}$$

where $\mu(\alpha,\beta)$ is the distributed function, $\gamma_{\alpha\beta}$ is the hysteresis operator, $z(t)$ is the monotonically increased or decreased position in the pre-sliding region and α and β correspond to up and down switching values of the input position, respectively. Figure 4 shows the friction force according to increase or decrease of the position. Also, Eq. (12) can be rewritten as follows:

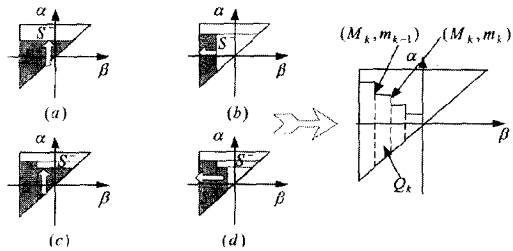


Fig. 4 Geometric analysis of the Preisach model

$$f_h(z)=\sum_{Q_k}\iint_{S^+}\mu(\alpha,\beta)\,d\alpha d\beta \tag{13}$$

where Q_k represents trapezoids in S^+ set, S^+ is positive set in Preisach plane.

When the input position is monotonically increased as shown in Fig. 4(a) and Fig. 4(c), Eq. (13) can be rewritten as follows:

$$\begin{aligned}f_h(z)=&\sum_{k=1}^{N}[F(\alpha_k,\beta_{k-1})-F(\alpha_k,\beta_k)]\\&+F(z(t),\beta_N)\end{aligned} \tag{14}$$

where

$$F(\alpha_k,\beta_k)=\iint_{Qk}\mu(\alpha,\beta)\,d\alpha d\beta$$

In the case of the monotonically decreasing input position, which is shown in Fig. 4(b) and Fig. 4(d), Eq. (13) can be rewritten as follows:

$$\begin{aligned}f_h(z)=&\sum_{k=1}^{N-1}[F(\alpha_k,\beta_{k-1})-F(\alpha_k,\beta_k)]\\&+F(\alpha_N,\beta_{N-1})-F(\alpha_N,z(t))\end{aligned} \tag{15}$$

Therefore, the hysteresis friction motion can be described by Eqs. (14) and (15). To complete the friction model based on the Preisach model, the values of $F(\alpha,\beta)$ should be known. To obtain them, however, a lot of experiments should be executed and it takes a long time. Furthermore, the friction motion of mechanical systems is not almost reappeared equally such as that of electro-magnetic systems. This is due to the irregularity of friction dependant on the contacting surface. Therefore, the reformed Dahl model should be used for obtaining the transition curve as follows (Awabdy, 1998):

$$f_{\dot{x}<0}=\frac{f_r(f_p+f_r)-sf_p(x_r-x)}{f_p+f_r+s(x_r-x)} \quad (16)$$

$$f_{\dot{x}>0}=\frac{-f_r(f_p+f_r)+sf_p(x_r+x)}{f_p+f_r+s(x_r+x)} \quad (17)$$

where

$$f_r=(f_s/2)\lfloor\{(1+k)^2+4k\}^{1/2}-(1+k)\rfloor$$

$$k=S\frac{x_r}{f_s}$$

and f_r is the friction force at the velocity reversal, f_p is the steady state rolling Coulomb friction force, x_r is the linear position at the velocity reversal, and s represents the initial and reversal slope of the hysteresis loop.

Using Eqs. (16) and (17), the main increasing and decreasing loops for the Preisach model are completed. Figures 5 and 6 show the results of the experimental and simulation hysteresis loops in the pre-sliding region of a ball-screw system. Based on it, the Preisach plane is accomplished.

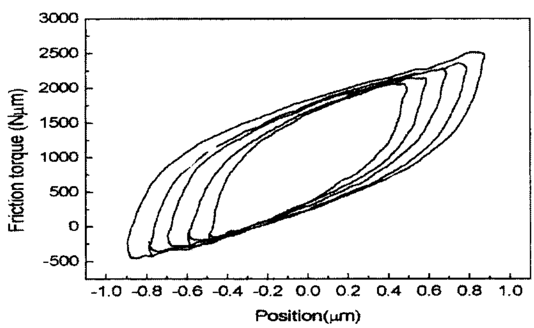


Fig. 5 Hysteresis phenomenon in the pre-sliding region

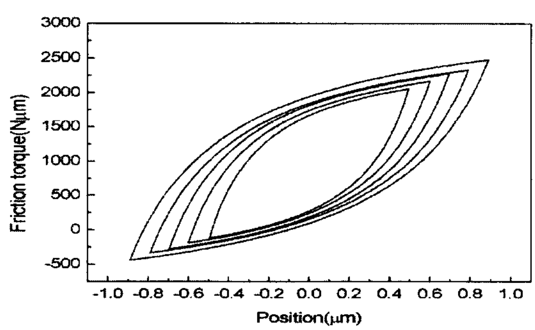


Fig. 6 Hysteresis phenomenon in the reformed Dahl Model

3. Design of the SMC for Precise Position Tracking Control

A ball-screw system is considered for precise tracking control whose dynamics is as follows:

$$J\ddot{\theta}=u-F_p \quad (18)$$

where J and θ are the inertia and the angle of the ball-screw, respectively and F_p is the friction force. The control scheme for precise position tracking of the ball-screw system is the SMC. To design the SMC, the sliding surface using error state is defined as follows:

$$S(t)=\dot{e}+\lambda e \quad (19)$$

where e is $\theta_d-\theta$ and $\lambda(>0)$ is a design parameter.

In order to induce the SMC which can satisfy the Lyapunov stability with the reaching condition, the Lyapunov candidate function is defined as follows (Erbatur, 1999):

$$V(t)=\frac{1}{2}s^2(t) \quad (20)$$

To satisfy the Lyapunov stability, the derivative of $s(t)$ is chosen as follows:

$$\dot{s}(t)=-K_1s(t)-K_2\,\mathrm{sgn}(s) \quad (21)$$

where K_1 and $K_2(>0)$ are design parameters. Then, the sliding motion is induced as follows:

$$\begin{aligned}\dot{s}(t)&=\ddot{e}(t)+\lambda\dot{e}=\ddot{\theta}_d-\ddot{\theta}(t)+\lambda\dot{e}\\&=\ddot{\theta}_d-\frac{1}{J}(u+F_P)+\lambda\dot{e}\end{aligned} \quad (22)$$

From Eqs. (21) and (22), the final sliding mode control law is selected as follows:

$$u=J\{\ddot{\theta}_d+\lambda\dot{e}+K_1s(t)+K_2\,\mathrm{sgn}(s(t))\}+F_P \quad (23)$$

The friction term in Eq. (23) is compensated with the feedforward term F_P using the proposed friction model for the pre-sliding region.

4. Experiment and Simulation

A ball-screw system is considered to show the friction phenomenon in the pre-sliding region.

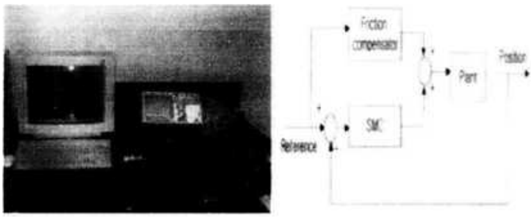


Fig. 7 The experimental layout and control scheme

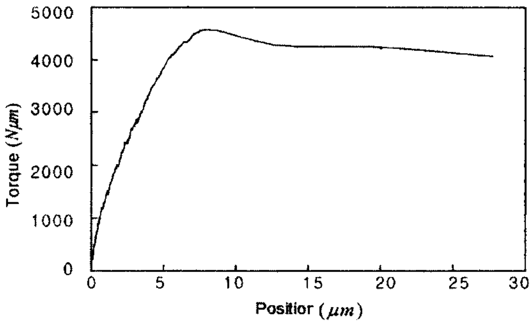


Fig. 8 Break-away torque against position

The ball-screw system has the linear guide and works on a DC servomotor with an encoder whose resolution is 10,000 pulse/rev. The torque was measured by the torque sensor of RSL-210 produced by the Lorenze Messtechnik GmbH company (Germany) which was attached between the motor and the ball-screw. The control system was implemented by the LabVIEW system of N.I. company. The experimental layout and control scheme are shown in Fig. 7. Firstly, to find the position changed from the pre-sliding to the sliding region, the friction torque was obtained at the ascending position, and the breaking torque appears around 7 μm. To show the wiping-out phenomenon, which is one of the representative characteristics of the non-local memory hysteresis, the command position input was excited to the ball-screw system which was moved forward and backward. The experimental and simulation results for this command input are shown in Figs. 9 and 10, respectively. The Preisach model used for simulation was accomplished using 100 transition branches of the hysteresis curve that is based on Eqs. (16) and (17). As shown in Figs. 9 and 10, the proposed friction model shows well the friction motion with

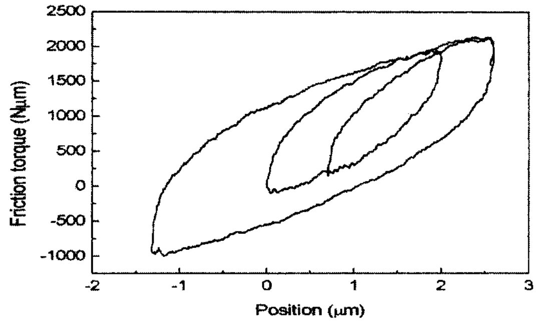


Fig. 9 Experimental result of the wiping-out phenomenon

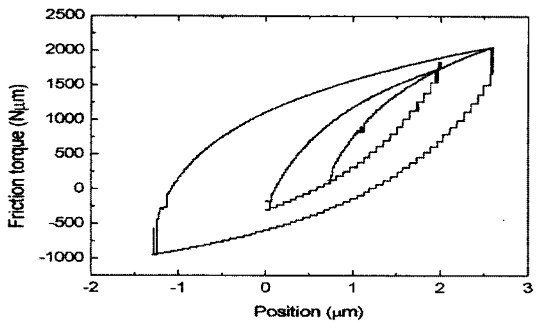


Fig. 10 Simulation result of the wiping-out phenomenon

the non-local memory characteristic in the pre-sliding region.

The precise position tracking control system was built by using the SMC and a feedforward friction compensator based on the proposed friction model. To evaluate the feedforward friction compensator a varied sine wave whose maximum magnitude is under 7 μm is used as command input. The tracking response of the ball-screw system is shown in Fig. 11, and the tracking position error is shown in Fig. 12. Peak to peak values of the error response for the SMC with friction compensator and without it are 0.69 μm and 1.46 μm, respectively, which means 53% reduction by using the friction compensator. In order to compare the results quantitatively, the distribution graphs for the error are shown in Fig. 13. As shown in figure, standard deviation and mean of SMC with friction compensator are from -0.2 to 0.9 μm and 0.15 μm, and from -0.4 μm to 1 μm and 2.3 μm are for without friction

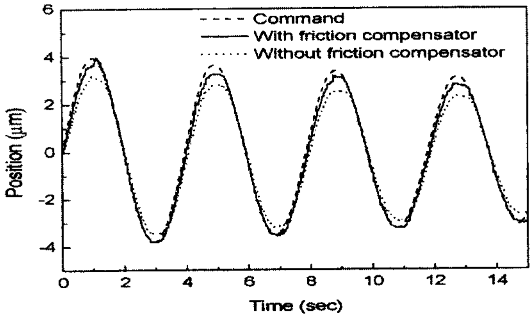


Fig. 11 Command input and position tracking results for the SMC with/without friction compensator

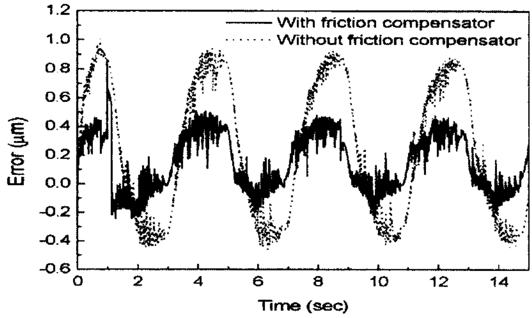
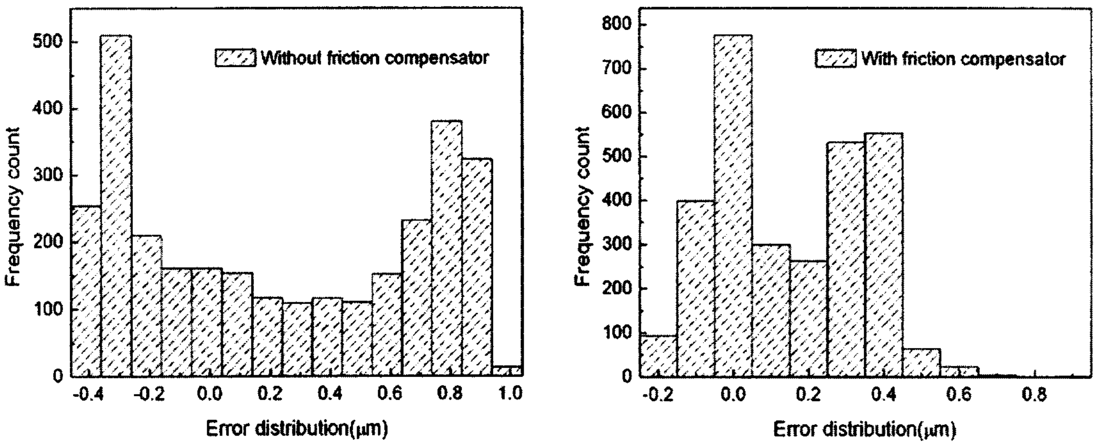


Fig. 12 Tracking error of the SMC with/without friction compensator



(a) Without friction compensator

(b) With friction compensator

Fig. 13 Standard deviation for the error

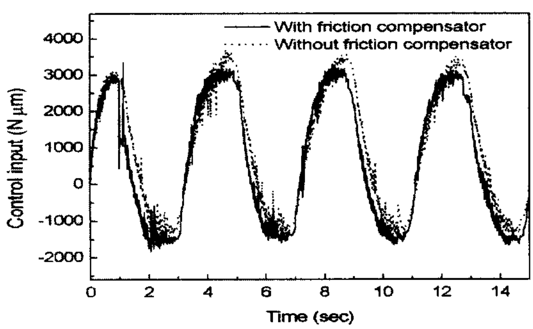


Fig. 14 Control inputs of the SMC with/without friction compensator

compensator.

As shown in the experimental results, the performance of the forward direction is worse than the backward direction, because the friction torque is not even forward and backward direction. However, the SMC with the proposed friction compensator has the better performance without excessive increasing of the control input as shown in Fig. 14.

5. Conclusion

The friction motion in the pre-sliding region was proposed by the Preisach hysteresis model. The proposed friction model is based on the LuGre model and supplements the pre-sliding friction motion. To apply the Preisach hysteresis model, the deformed Dahl model is used. By using the deformed Dahl model, the number and time for experiment could be reduced and the satisfactory results were obtained. Also, based on the proposed friction model, the friction compensation control scheme in the pre-sliding region was proposed by the SMC with feedforward hysteresis compensator. Through experiments, it was found that the SMC with feedforward hysteresis compensator had better performance than the SMC without compensator.

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